رياضيات هندسية (1)

تم رفع بواسطة
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First

56/100

Palestine Technical University
Mathematics Department.
Linear Algebra and differential Equations.
First Exam

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Part I (45 points): Determine whether each statement is **True** or **False.**

1. (F)If $N(A)$=**0**, then the matrix A is nonsingular.
2. (T)Suppose A and B are two nxn nonsingular matrices, then $A+B^{-1}$ is nonsingular.
3. (F)The matrix $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in reduced row echelon form.
4. (T)The set $S=\{(x,y,z)\in R^3 : yz=0\}$ is a subspace of R^3.
5. (T)If ~~A is~~ nonsingular, then $adj(A)$ is nonsingular.
6. (F)If A and B are symmetric, then AB is symmetric.
7. (F)If A and B are two nxn matrices, then $(A+B)^2=A^2+2AB+B^2$
8. (T)If $AB=I_n$, then $det(A)\neq 0$ and $det(B)\neq 0$.
9. (F)A homogenous system of three equations and four unknowns has a nontrivial solution.
10. (T)There are vector spaces with 10 elements.
11. (T)If A is nonsingular, then A^T, $2A$ and A^{-1} are all nonsingular.
12. (F)If A is obtained from B by applying three elementary row operations on B, then $det(A)=det(B)$.
13. (T)The matrix $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is nonsingular. $\cos^2+\sin^2$
14. (F)If A is a non singular symmetric matrix, then $A^{-1}=(A^{-1})^T$.
15. (T)If A is symmetric, then A^T symmetric.

$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = -2$

/27

Part II

1- Let $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 3\\ 5 & 5 & 1\end{bmatrix}$. (15 points.)

a. Use reduction to compute A^{-1}.

$$\left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 2 & 3 & 0 & 1 & 0\\ 5 & 5 & 1 & 0 & 0 & 1\end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0\\ 5 & 5 & 1 & 0 & 0 & 1\end{array}\right]$$

$$\xrightarrow{-5R_1+R_3} \left[\begin{array}{ccc|ccc}1 & 1 & 1 & 1 & 0 & 0\\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0\\ 0 & 0 & -4 & -5 & 0 & 1\end{array}\right] \xrightarrow{-1R_2+R_1} \left[\begin{array}{ccc|ccc}1 & 0 & \frac{-1}{2} & 1 & -\frac{1}{2} & 0\\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0\\ 0 & 0 & -4 & 0 & 0 & 1\end{array}\right] \xrightarrow{-\frac{1}{4}R_3}$$

$$\left[\begin{array}{ccc|ccc}1 & 0 & \frac{-1}{3} & 1 & \frac{-1}{2} & 0\\ 0 & 1 & \frac{3}{2} & 0 & \frac{1}{2} & 0\\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{4}\end{array}\right] \xrightarrow{R_2=-\frac{3}{2}R_3+R_2} \left[\begin{array}{ccc|ccc}1 & 0 & \frac{-1}{3} & 1 & \frac{-1}{2} & 0\\ 0 & 1 & 0 & 0 & \frac{1}{2} & \frac{3}{8}\\ 0 & 0 & 1 & 0 & 0 & \frac{-1}{4}\end{array}\right] \rightarrow$$

$$R_1=\frac{1}{3}R_3+R_1 \rightarrow \left[\begin{array}{ccc|ccc}1 & 0 & 0 & 1 & \frac{-1}{2} & \frac{-1}{12}\\ 0 & 1 & 0 & 0 & \frac{1}{2} & \frac{3}{8}\\ 0 & 0 & 1 & 0 & 0 & \frac{-1}{4}\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}1 & \frac{-1}{2} & \frac{-1}{12}\\ 0 & \frac{1}{2} & \frac{3}{8}\\ 0 & 0 & -\frac{1}{4}\end{bmatrix}$$

b. Use your answer in part a to solve the system $Ax=b$, where $b=\begin{pmatrix}8\\24\\6\end{pmatrix}$

$$\begin{bmatrix}1 & \frac{-1}{2} & \frac{-1}{12}\\ 0 & \frac{1}{2} & \frac{3}{8}\\ 0 & 0 & \frac{-1}{4}\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}8\\24\\6\end{bmatrix}$$

$x = A^{-1}b$

$\Rightarrow x_1-\frac{1}{2}x_2-\frac{1}{12}x_3=8$

$\frac{1}{2}x_2+\frac{3}{8}x_3=24$

$\frac{-1}{4}x_3=6 \Rightarrow \boxed{x_3=-24}$ -0

$\frac{1}{2}x_2+\frac{3}{8}(-24)=24$

$\frac{1}{2}x_2+-9=24 \Rightarrow \frac{1}{2}x_2=33 \Rightarrow \boxed{x_2=66}$

$x_1-\frac{1}{2}(66)-\frac{1}{12}(-24)=8$

$x_1-33+2=8$

$x_1-31=8$

$x_1=39$

$\boxed{x_1=39}$

$\boxed{x_2=66}$

$\boxed{x_3=-24}$

10

2- Consider the linear system (20 points.)

$$\begin{aligned} x + y - z &= 2 \\ x + 2y + z &= 3 \\ x + y + (a^2 - 5)z &= a. \end{aligned}$$

a) Find all values of a for which the resulting linear system has

a. no solution.

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 1 & 2 & 0 & 3 \\ 1 & 1 & (a^2-5) & a \end{array}\right] \quad R_2 = -R_1 + R_2 = \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & (a^2-5) & a \end{array}\right]$$

$$R_3 = -1R_1 + R_2 \left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & (a^2-5) & a-2 \end{array}\right]$$

for the System has No Solution

$a^2 = 5 \Rightarrow a = \pm\sqrt{5}$ / $a \neq 2$

b. a unique solution.

has a unique solution when $a \neq \pm\sqrt{5}$ ~~and $a \neq 2$~~

c. infintely many solutions.

for infintely Many Solution

$a = \pm\sqrt{5}$ & $a = 2$

17

4. Determine whether the vectors $v=(-1,4,2,2)^T$ *and* $w=(1,-1,2,0)^T$ belong to Span ($v_1=(1,0,0,1)^T, v_2=(1,-1,0,0)^T, v_3=(0,1,2,1)^T$) (20 points.)

$$v = \begin{pmatrix} -1 \\ 4 \\ 2 \\ 2 \end{pmatrix} \quad w = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix}$$

$$\alpha_1 \begin{pmatrix} -1 \\ 4 \\ 2 \\ 2 \end{pmatrix} + \alpha_2 \begin{pmatrix} 1 \\ -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

X

$$\begin{pmatrix} -\alpha_1 \\ 4\alpha_1 \\ 2\alpha_1 \\ 2\alpha_1 \end{pmatrix} + \begin{pmatrix} \alpha_2 \\ -\alpha_2 \\ 2\alpha_2 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & a \\ 4 & -1 & b \\ 2 & 2 & c \\ 2 & 0 & d \end{array}\right] \xRightarrow{R_1 = -R_1} \left[\begin{array}{cc|c} 1 & -1 & -a \\ 4 & -1 & b \\ 2 & 2 & c \\ 2 & 0 & d \end{array}\right]$$

$$R_2 = -4R_1 + R_2 = \left[\begin{array}{cc|c} 1 & -1 & -a \\ 0 & 3 & 4a+b \\ 2 & 2 & c \\ 2 & 0 & d \end{array}\right] \quad R_3 = -2R_1 + R_3 =$$

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$$\left[\begin{array}{cc|c} \cancel{1} & \cancel{-1} & -a \\ 0 & \cancel{3} & \cancel{4a+b} \\ \cancel{0} & \cancel{4} & \cancel{2a+c} \\ \cancel{2} & \cancel{0} & \cancel{d} \end{array}\right] \quad \cancel{R_3} =$$

$\cancel{R_3 \leftrightarrow R_4}$

$$\left[\begin{array}{cc|c} \cancel{1} & \cancel{1} & -a \\ 0 & 3 & 4a+b \\ 2 & 0 & d \\ 2 & 2 & c \end{array}\right] \Rightarrow \cancel{R_3 = -2R_1 + R_3} = \left[\begin{array}{cc|c} \cancel{1} & \cancel{-1} & \cancel{-a} \\ \cancel{0} & \cancel{3} & 4a+b \\ \cancel{0} & \cancel{2} & \\ \cancel{2} & & \end{array}\right]$$

2

$$\cancel{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 5 & 1 & 2 \\ 6 & 2 & 4 \end{bmatrix}}$$

$$\cancel{\begin{bmatrix} 4+10+18, & 5+2+6, & 6+4+12 \\ 8+5+12 & & \end{bmatrix}} \quad \cancel{\begin{bmatrix} 32 & 13 & 22 \\ 25 & 15 & 22 \end{bmatrix}}$$

$$R_2 = -4R_1 + R_2 = \left[\begin{array}{cc|c} 1 & -1 & -9 \\ 0 & 3 & 4a+b \\ 2 & 2 & c \\ 2 & 0 & d \end{array}\right]$$

$2 = \frac{c}{c}$

$3 =$

$$R_3 = -2R_1 + R_3 = \left[\begin{array}{cc|c} 1 & -1 & -a \\ 0 & 3 & 4a+b \\ 0 & 4 & 2a+c \\ 2 & 0 & d \end{array}\right]$$

$$\Rightarrow R_4 = -2R_1 + R_4 = \left[\begin{array}{cc|c} 1 & -1 & -9 \\ 0 & 3 & 4a+b \\ 0 & 4 & 2a+c \\ 0 & 2 & 4a+b \end{array}\right]$$

v & w are ~~not~~ ~~span~~

Spaning